Are Minimum Wages and Income Taxes Complements or Substitutes?*

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Abstract

In this paper, we document that minimum wages and income taxes are positively correlated in some U.S. states, and negatively correlated in others. This fact cannot be explained by Ramsey outcomes under complete information. However, the optimal public policy under asymmetric information can rationalize this fact. In particular, we show that the optimal minimum wage and the optimal income tax are complements in a frictionless economy. When workers privately observe their productivity, in contrast, we find that the two redistributive tools can be substitutes under plausible restrictions on the government's auditing technology.

Keywords: Optimal taxation, minimum wage, asymmetric information, skill-biased technical change, tax evasion

JEL Classification Numbers: D82, H21, H26, J31, J38

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1 Introduction

Modern governments possess a variety of policy instruments that can be used to redistribute income. Among these, the minimum wage and the income tax have been the source of much investigation. However, little consideration has been given to the *interaction* between these two redistributive policies.

The contributions of this paper are both empirical and theoretical. First, we apply standard techniques from the industrial organization literature to empirically investigate the interaction between the minimum wage and the income tax.[1] Using state-level data from the U.S. over the period 2000-2015, we document that the minimum wage and the (maximum) income tax are complements in some U.S. states, and substitutes in others. We also find that that there is significant heterogeneity in the correlation between these two policy instruments over time.

We then embed a general equilibrium model in a contracting framework with hidden types to account for this fact. In particular, we consider an environment comprised of three types of agents: workers heterogenous in productivity, a representative firm, and a government. We assume that the only source of exogenous variation are changes in the skill distribution, which are privately observed by workers. To alleviate this information asymmetry, the government can access an auditing technology which is dependent on public policy. We focus on the Maximin social objective and endow the government with two redistributive tools to maximize this objective. In particular, it can set a wage floor and a linear income tax supported with a lump-sum transfer. The use of either instrument evokes the familiar trade-off between equity and efficiency.

The main comparative statics exercise of interest is how public policy responds to skill-biased technical change (SBTC), which is a shift in the production technology that favors high-skilled labor over low-skilled labor by increasing its relative productivity. Ceteris paribus, SBTC induces a rise in the skill premium: the ratio of high-skilled to low-skilled wages (for surveys, see Acemoglu, 2002; Aghion, 2002; Hornstein et al., 2005). To be consistent with our analysis, we tweak the notion of the skill premium that is traditionally used in the labor literature. As opposed to comparing the skills of workers without a college degree to the skills of workers with a college degree, we contrast the skills of workers earning the minimum wage to the rest of the labor force. Henceforth, we refer to this measure as the Minimum Wage-adjusted Skill Premium (MWASP). We find that changes in the MWASP can explain much of the recent variation in redistributive policy.

Theoretically, we show that in response to observable skill shocks, the Ramsey planner

[1]Economists in the field of industrial organization have long been interested in testing the interaction between different practices (Arora and Gambardella, 1990; Athey and Stern, 1998; Carree et al., 2011). In this context, complementarity is said to exist between two practices if the implementation of one practice amplifies the marginal return on the other.

finds it optimal to set a public policy in which the minimum wage and the income tax are always complementary. In contrast, when workers privately observe their productivity, we find that the optimal minimum wage and the optimal income tax can be substitutes under realistic restrictions on the governments auditing technology. The key assumption that allows us to derive a malleable relationship between the policy instruments is that government regulations that reduce disposable income perpetuate misreporting of productivity. We provide various pieces of evidence from the literature that support this assumption. When information is complete, public policy is set merely to balance efficiency losses with redistributive gains. We show that irrespective of whether public policy is independently or jointly determined, income taxes and minimum wages are used to address these two motives in a complementary fashion. But when workers privately observe their productivity and are more enticed to misreport these shocks when their disposable income is low, increasing the income tax rate or reducing the minimum wage increases the value of misreporting information. To prevent this deviation, a social planner may find it desirable to construct a public policy in which minimum wages and income taxes are substitutes.

On a technical note, a novelty of our approach is that we cast the social planner's problem in a manner that permits us to apply supermodular optimization theory to tease out the relationship between the policy instruments.

Lastly, we quantitatively illustrate the main findings of the paper in a model disciplined by U.S. data. The period of interest witnessed a steady increase in the skill premium. We feed this trend into the model, and parametrically estimate auditing technologies in each state to match the correlations between the minimum wage and the income tax observed in the data.

Related Literature: This paper is related to a relatively small normative literature in public economics which investigates if the minimum wage is desirable for redistributive reasons on top of optimal taxes and transfers. This literature has mostly focused on the Stiglitz (1982) model with two skills and endogenous and competitive wages and has found that a minimum wage is helpful to supplement the optimal linear tax system (Allen, 1987; Guesnerie and Roberts, 1987).[2] These findings are at odds with the data. In particular, these frameworks cannot account for the substitution between the minimum wage and the income tax that we observe in some U.S. states. From a positive perspective, virtually no attention is devoted to the interaction between the minimum wage and the income tax, even though the literature that studies the incidence of these two redistributive instruments in isolation is vast.[3]

[2]Lee and Saez (2012) extend this analysis to study the case of non-linear taxation and show that the minimum wage and subsidies for low-skilled workers are complementary policies.

[3]See Kotlikoff and Summers (1987) for a survey on tax incidence, and Belman and Wolfson (2014) for a review on employment effects of the minimum wage.

2 Empirical Estimates

In this section, we document that minimum wages and income taxes are complements in some U.S. states, and substitutes in others. We use two standard approaches to test for complementarity. The first approach is based on revealed preference, while the second approach builds on the empirical production literature. Using time-series data, we also show that the two policy instruments positively co-move only over specific time-periods, as well as present evidence that suggests that SBTC is the relevant shock to consider when accounting for variations in income taxes and minimum wages.

2.1 Data

Federal and State Minimum Wages. Wage data for the years 2000 through the present was obtained from the U.S. Department of Labor, Office of State Standards Programs Wage and Hour Division web site Minimum Wage and Overtime Pay Standards Applicable to Nonsupervisory NONFARM Private Sector Employment Under State and Federal Laws. We make two exclusions. First, we discard wage floors that were imposed in terms of weekly units. Second, different wage floors are imposed in different industrial sectors in certain years. In such cases, we record the upper limit across sectors. For the empirical exercises that follow, we consider the *effective* minimum wage, i.e. the maximum value between the federal minimum wage and the state minimum wage.

Income Taxes. We consider the maximum state income tax rates from the NBER TAXSIM database.[4] In particular, we use the state rate for wages.

The sample period is 2000 to 2015. Data on federal and state minimum wages are also available for the years 1968 through 1998 in the Book of the States, 1968-1999 edition, volume 32 published by the Council of State Governments. However, this data has gaps. As we are also interested in first differences, we restrict attention to the uninterrupted time-series from 2000 onwards. Moreover, data on maximum state income tax rates is available from 1977 to 2015, which restricts the upper limit of the sample. 10 states feature zero variation in one of the indicators over this period.

2.2 Approach 1: Correlation

To measure complementarity, we follow Holmstrom and Milgrom (1994) and Arora and Gambardella (1990). Complementarity creates a force in favor of a positive correlation even

[4]See Feenberg and Coutts (1993) for more information.

after controlling for observable, exogenous characteristics.[5]

2.2.1 Levels

First, we show that there is a positive correlation between the minimum wage and income taxation. Figure 1 plots the correlation between levels of the effective minimum wage and the top wage income tax rate across U.S. states. The figure reveals that changes in the effective minimum wage and changes in the maximum state income tax are negatively correlated on aggregate when equal weight is allotted to each observation. Out of the 41 states that feature variability, there are 14 states with positive correlations and 27 states with negative correlations.

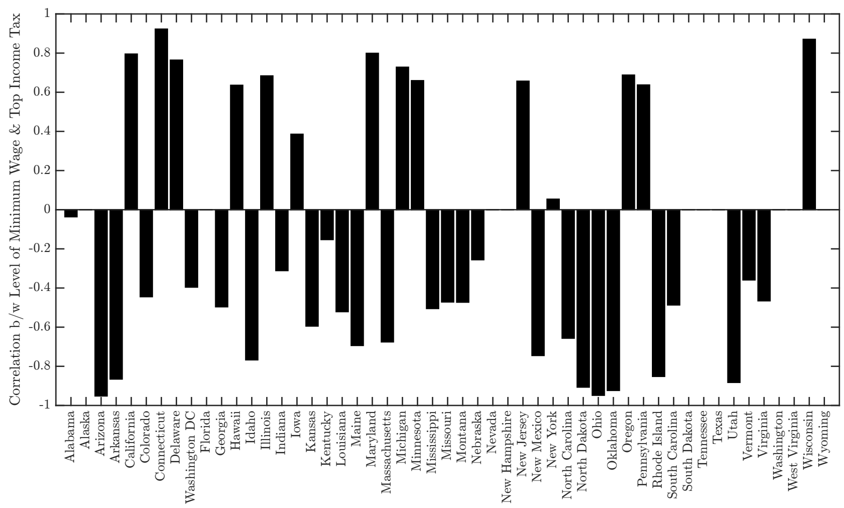


Figure 1: Correlations b/w the Minimum Wages and Maximum State Income Tax Rates

2.2.2 First Differences

Next, we show that there is a positive correlation also between changes in the minimum wage and changes in income taxation. Figure 2 plots the correlation between first differences in the effective minimum wage and first differences in the top wage income tax rate across U.S. states. The figure reveals that changes in the effective minimum wage and changes in the maximum state income tax are negatively correlated on aggregate when equal weight is allotted to each observation. Out of the 41 states that feature variability, there are 13 states with positive correlations and 28 states with negative correlations.

[5] Arora and Gambardella (1990) show that any two strategies are complementary if they are positively correlated. Other work focused on testing for complementarity includes Brickley (1995), Brynjolfsson and Hitt (1998), Colombo and Mosconi (1995), Greenan and Guellec (1998), Ichniowski et al. (1997), MacDuffie (1995), and Pil and MacDuffie (1996).

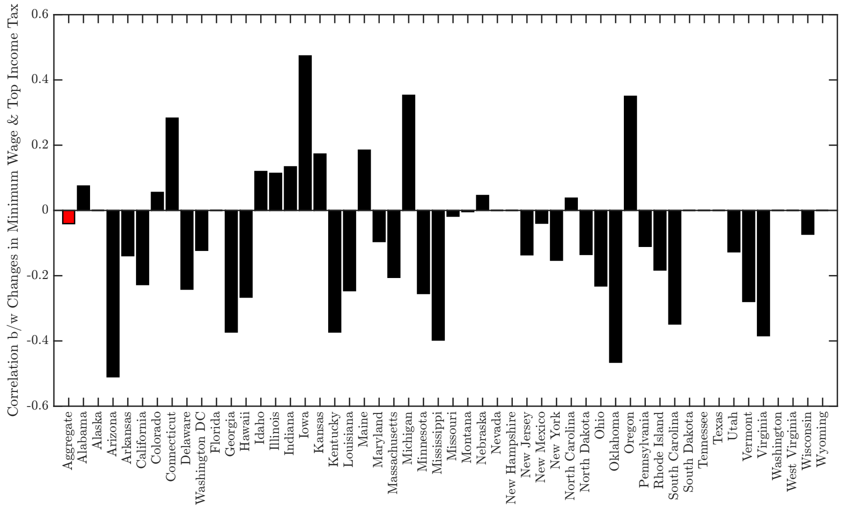


Figure 2: Correlations b/w Changes in Minimum Wages and Maximum State Income Tax Rates

2.3 Approach 2: Regression

The second main approach to test for complementarity has been to build on the empirical production literature. This approach relies on an OLS regression of a measure of welfare on a set of regressors, including interaction effects between different practices.

2.3.1 Time Units

We first test for complementary by considering state-specific specifications. In particular, we consider the following specification:

$$y_s = \alpha_s + \beta_{\tau,s}\tau_s + \beta_{\bar{w},s}\bar{w}_s + \beta_{\pi,s}\pi_s + \beta_{\tau,\bar{w},s}\tau_s \times \bar{w}_s + \beta_{\tau,\pi,s}\tau_s \times \pi_s + \beta_{\bar{w},\pi,s}\bar{w}_s \times \pi_s + \beta_{\tau,\bar{w},\pi,s}\tau_s \times \bar{w}_s \times \pi_s + \epsilon_s,$$

where $\epsilon_s \sim \mathcal{N}(0, \sigma_s^2)$. Here s represents a U.S. state, y represents log levels of Real GDP, τ represents the maximum state income tax, $\bar{w}$ represents the minimum wage, and π captures the implied GDP Deflator. The test for complementarity holds only when the interaction term is non-negative. To respect this constraint, we use a price index instead of growth rates. In particular, the GDP Deflator is indexed to 1 in 2009. For each state s, notice that the complementarity between the minimum wage and the income tax depends on the cross-derivative

$$\frac{\partial^2 y_s}{\partial \tau_s \partial \bar{w}_s} = \beta_{\tau,\bar{w},s} + \beta_{\tau,\bar{w},\pi,s} \times \pi_s.$$

If $\pi_s \geq 0 \ \forall s$, then the minimum wage and the income tax are complementary if $\beta_{\tau,\bar{w},s} \geq 0$ and $\beta_{\tau,\bar{w},\pi,s} \geq 0$ with at least one of the inequalities holding strictly. Due to the small sample size, i.e. 16 observations per state, statistical significance is hard to achieve. Yet, even at the 95 percent confidence level, Arizona, Hawaii, Illinois, Indiana, New Jersey, Utah and Wisconsin fail this criteria. A weaker restriction for the minimum wage and the income tax to be complements is that $\beta_{\tau,\bar{w},s} + \min \pi_s \beta_{\tau,\bar{w},\pi,s} \geq 0$ and $\beta_{\tau,\bar{w},s} + \max \pi_s \beta_{\tau,\bar{w},\pi,s} \geq 0$ with at least one of the inequalities holding strictly. Figure 3 reports point estimates for the two expressions of interest across U.S. states.

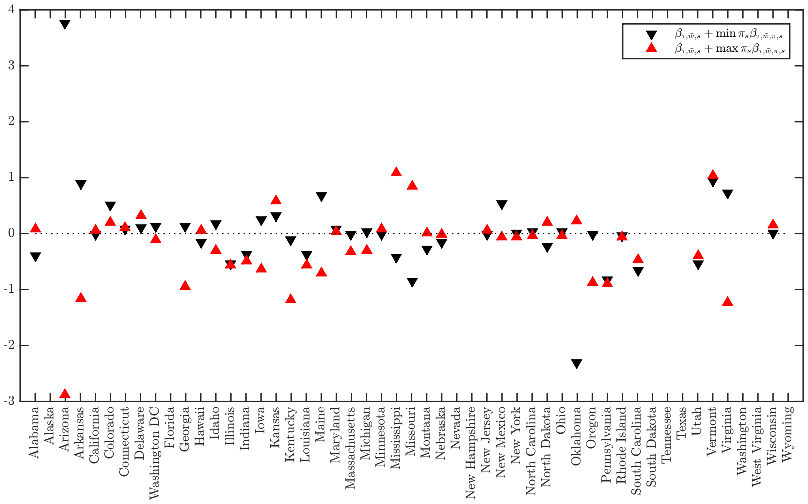


Figure 3: Testing for Complementarity across U.S. States

2.3.2 Location-time Units

This section uses data in state-time units to obtain finer estimates of the complementarity between the redistributive policy instruments. The baseline specification adapts merely the regression used to study within-state variations, by equalizing coefficients across states. Without carefully controlling for exogenous variations across states, this procedure is susceptible to biased estimates.

We follow the growth literature (Levine and Renelt, 1992) in choosing the appropriate controls. Researchers in this field focus mainly on cross-country analyses, often exploiting variations in variables such as the investment share of GDP and the ratio of exports to GDP. However, real sector data at the state-level is available only from the production side, which inhibits us from controlling for these variables at the state level. Credit growth is also a factor that this literature identifies as essential for economic growth. For these variables, we control for national level data that we obtain from the FRED Database. There is another set of variables identified by the literature as pivotal for growth and for which data is available at the state level. This set of variables includes population levels, which are compiled by the U.S. Census, and government production, which are compiled by the Bureau of Economic Analysis.[6]

[6]The growth literature typically exploits data from the expenditure side on government consumption. These estimates are unavailable at the state level, due to which we employ data from the production side.

Table 1 reports the point estimates for the coefficients of interest for five different model specifications. Column (1) captures the baseline specification without any controls. Specification (2) augments specification (1) by controlling for idiosyncratic population levels. Specification (3) augments specification (2) by controlling for state-specific production by government enterprises. Specification (4) augments specification (3) by controlling for real sector variables at the aggregate level, i.e. investment share of GDP and export share of GDP. Specification (5) augments specification (4) by controlling for changes in monetary conditions, i.e. growth in total credit to private non-financial sector (adjusted for breaks).

Observe that all specifications reject the hypothesis of complementarity. In particular, in all specifications $\beta_{\tau,\bar{w}} < 0$, i.e. the minimum wage and the income tax fail the strict criterion for complementarity as it requires $\beta_{\tau,\bar{w}}$ to be non-negative. Moreover, in all specifications $\beta_{\tau,\bar{w}} + \min \pi \beta_{\tau,\bar{w},\pi} < 0$. Thus, the minimum wage and the income tax also fail the weak criterion for complementarity which requires $\beta_{\tau,\bar{w}} + \min \pi \beta_{\tau,\bar{w},\pi}$ to be non-negative.

Table 1: Testing for Complementarity in Location-time Units

	(1)	(2)	(3)	(4)	(5)
$\beta_{\tau,\bar{w}}$	-0.1838	-0.2148***	-0.2188***	-0.2195***	-0.2189***
	(0.1173)	(0.0601)	(0.0596)	(0.0596)	(0.0596)
$\beta_{\tau,\bar{w},\pi}$	0.1748	0.1461**	0.1523**	0.154***	0.153***
	(0.1167)	(0.0599)	(0.0593)	(0.0594)	(0.0594)
$\beta_{\tau,\bar{w}} + \min \pi \beta_{\tau,\bar{w},\pi}$	-0.0631	-0.1139	-0.1136	-0.1132	-0.1132
$\beta_{\tau,\bar{w}} + \max \pi \beta_{\tau,\bar{w},\pi}$	0.025	-0.0403	-0.0369	-0.0356	-0.0361
R-squared	0.0187	0.7424	0.7476	0.748	0.748
F-statistic	2.197	290.7	265.2	216.9	198.7
P-value	0.0326	< 0.0001	< 0.0001	< 0.0001	< 0.0001

Notes: Column (1) captures the baseline specification without any controls. Specification (2) augments specification (1) by controlling for idiosyncratic population levels. Specification (3) augments specification (2) by controlling for state-specific production by government enterprises. Specification (4) augments specification (3) by controlling for the investment share of GDP and export share of GDP. Specification (5) augments specification (4) by controlling for the growth in total credit to private non-financial sector (adjusted for breaks). ***, ** and * notate that the two-sided p-value < 0.01, 0.05, and 0.1 respectively; standard errors are reported in brackets. The two-sided p-value < 0.15 for specification (1).
Source: Bureau of Economic Analysis, FRED, NBER TAXSIM, U.S. Census, U.S. Department of Labor, and author's calculations.

2.4 SBTC and Public Policy

One approach to show that a SBTC is the relevant shock to consider is to compare changes in the policy instruments to changes in the skill premium. In particular, we could ask if episodes in which the skill premium changed coincide with episodes in which the policy instruments also varied. As discussed in the following sections, in general, such an analysis can be misleading as the information set confounds the relationship between SBTC and the optimal public policy. When workers privately observe their productivity, we find that optimal public policy may be unperturbed even in the event of a SBTC. To circumspect this issue, we restrict attention to the time period in which the skill premium breached the critical threshold beyond which incentive constraints are estimated to be slack, i.e. 2007 to 2015; see section 5.3.

We first establish that the correlation between the policy instruments increased substantially post-2006. The correlation between the policy instruments pre-2006 was -0.14, while the correlation between the policy instruments post-2006 was 0.4.

We then exploit variation at the state level by running a two-sample t-test to show that the mean levels of policy instruments pre-2006 are significantly different than those post-2006. In the case of both policy instruments, we find that the t-test rejects the null hypothesis that the data comes from independent random samples from normal distributions with equal means; see Table 2.

Table 2: T-test for Pre- and Post-2006 Samples

	τ		$\bar{w}$	
	(1)	(2)	(3)	(4)
Confidence Interval	1.33	2.91	-1.88	-1.68
T-statistic	5.25		-34.07	
Degrees of Freedom	814		814	
Population Standard Deviation	5.72		0.74	

Notes: This table reports the 95% confidence intervals for the difference in population means of taxation and minimum wages over the period 2000-2006 and 2007-2015. Columns (1) and (3) report lower bounds, while columns (2) and (4) report upper bounds.

Lastly, we measure the extent of the variability in policy instruments that can be accounted for by changes in the skill premium post-2006. In particular, we regress each policy variable (in state-time units) on the MWASP, documenting a significant relationship between the two at the 1% confidence level. A 100 basis point increase in the MWASP is estimated to increase (overall) taxation by 9.6 percentage points, and increase minimum wages by 2.5 USD; see Figure 4. The R-squared for the regression with tax as the independent variable is 0.27, and the R-squared for the regression with minimum wages as the independent variable is 0.59. These findings suggest that variations in the skill premium can account for a substantial proportion of the change in taxation and minimum wages.[7]

In summary, there is a significant degree of heterogeneity in the correlation between minimum wages and income taxes across states and over time. Moreover, much of the variation in these policy instruments can be explained by SBTC. As we argue in the following section, these observations cannot be rationalized by optimal public policy in a frictionless economy.

[7]See Tyson and Mendonca (2018) for recent commentary on how the minimum wage and the Earned Income Tax Credit (EITC) are regarded as among the most effective tools to tackle the rise in the skill premium.

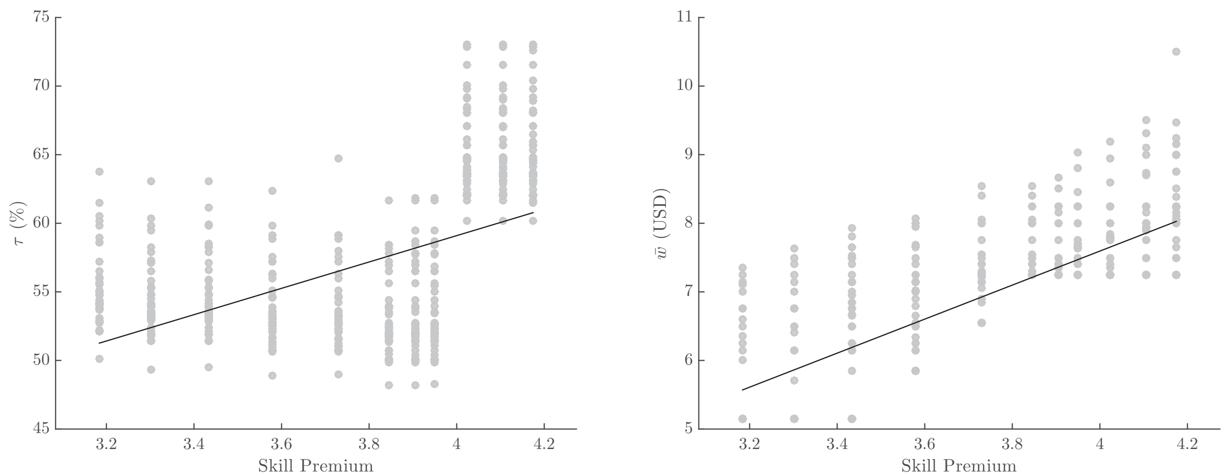


Figure 4: Relationship between Policy Variables and Skill Premium Post-2006

3 Model

This section spells out the environment, defines the equilibrium, and presents the planning problem. We consider a stylized static setting where production requires two labor inputs. The model is comprised of three types of agents: workers heterogenous in productivity, a representative firm, and a government. The primary motivation for the assumed form of preferences and technology is that it will allow us to characterize the social welfare function in a manner that is conducive to the use of the theory of supermodular optimization.

Firms

Production of a single consumption good $F(h_L, h_H)$ depends on the number of low-skilled workers h_L and the number of high-skilled workers h_H. We assume that the production function is additively separable in its arguments. In particular, a worker with skill i can produce output using technology $F_i : \mathbb{R}_+ \mapsto \mathbb{R}_+$, where $F_i(x) = \theta_i x^\alpha$, $\alpha \in (0, 1)$, $\theta_i > 0$. We assume $\theta_H > \theta_L$. The production section chooses labor demand (h_L, h_H) to maximize profits: $\Pi = F_L(h_L) + F_H(h_H) - w_L h_L - w_H h_H$.

Households

We assume each individual is either low-skilled or high-skilled. We normalize the population of workers to one and denote by μ_L and μ_H the fraction of low-skilled and high-skilled workers respectively with $\mu_L + \mu_H = 1$. Individual with skill i makes a consumption decision, c_i, and labor decision, l_i, to maximize utility $u(c, l) = \log(c) - \chi l$, $\chi > 0$. Individuals receive utility from consuming and disutility from work; in particular, $\partial u/\partial c_i > 0$, $\partial u/\partial l_i < 0$ and u is concave. Individuals face a before tax wage of w_i, a linear labor tax of τ, and are supported by a government transfer T, which give rise to the following budget constraints: $c_i \leq w_i l_i(1 - \tau) + T \ \forall i \in \{L, H\}$.

Labor Market Clearing

We assume that the market for high-skilled workers behaves competitively, while employers have market power over low-skilled workers. In particular, small wage cuts below the competitive level do not induce low-skilled workers to leave the firm. We assume that the government can set a wage floor, $\bar{w}$, which may prevent the labor market from clearing. In particular, when $\bar{w} < w_i$, then wages are determined by the undistorted labor market clearing condition, $h_i = l_i$. However, when $\bar{w} \geq w_i$, then the equilibrium labor allocation is determined by the labor demand condition, $\frac{\partial F_i}{\partial h} = \bar{w}$.

Government Budget Constraint

The government balances its budget in that it finances lump-transfers $T \in \mathbb{R}_+$ completely using linear income taxation $\tau \in [0, 1]$. In particular, the government budget constraint is given by

$$\tau \sum_{i \in \{L,H\}} \mu_i w_i l_i = T. \tag{1}$$

How restrictive is the assumption of a flat income tax? Friedman (1962) argued for a negative income tax, which effectively combines a lump-sum transfer with a constant marginal tax rate.[8] Mirrlees (1971) pioneered the study of optimal non-linear tax schedules in environments with unobservable labor income risk. He did not impose any constraints on the shape of the tax schedule, but still found that the optimal schedule is close to linear. Moreover, since u is weakly separable in consumption and labor, the uniform commodity taxation result holds (Atkinson and Stiglitz, 1976). This result implies that commodity taxation is a suboptimal instrument to redistribute income, and redistribution should be carried out only through labor taxation. These findings suggest that an affine labor tax is close to optimal.

3.1 Competitive Equilibrium

Let $\mathcal{P} = (\tau, T, \bar{w}) \in [0, 1] \times \mathbb{R}^2_+$ denote the policy vector.

Definition 1. *A $\mathcal{P}$-distorted competitive equilibrium is $\{c_i, l_i, h_i, w_i\}_{i \in \{L,H\}}$:*
(i) Given $\{w_i, \tau, T\}$, $\{c_i, l_i\}$ solves household i's problem.

[8] This is manifested in the Hall–Rabushka flat tax proposal.

(ii) Given $\{w_i\}_{i\in\{L,H\}}$, $\{h_i\}_{i\in\{L,H\}}$ solves the firms' problem.
(iii) $\{l_i, h_i\}_{i\in\{L,H\}}$ are consistent with the minimum wage policy $\bar{w}$.
(iv) The government budget constraint holds (1) holds.

Let $\{c_i^*(.), l_i^*(.), h_i^*(.), w_i^*(.)\}_{i\in\{L,H\}}$ denote competitive equilibrium allocations, where the equilibrium functions map from $[0,1]\times\mathbb{R}_+^2$ to $\mathbb{R}_+$.

3.2 Planning Problem

Denote the Pareto weight of type i by ρ_i, and denote the governments information set by Ω. In the sequel, we focus on the solution to the following planning problem:

$$\max_{\mathcal{P}} \sum_{i\in\{L,H\}} \rho_i u(c_i{}^*(\mathcal{P}), l_i{}^*(\mathcal{P})) \quad \text{s.t.} \quad (c_i{}^*(\mathcal{P}), l_i{}^*(\mathcal{P})) \in \Omega.$$

4 Theoretical Analysis

In this section, we analytically characterize how the minimum wage and the income tax interact in the event of a skill-biased technical change. Studying a planning problem subject to implementability constraints (which guarantee that price-allocation pairs satisfy a competitive equilibrium) and incentive constraints (which guarantee that workers truthfully report their productivity) is technically challenging. We make progress by isolating restrictions on preferences and technology that allow the full information (Ramsey) problem to be re-written as an unconstrained maximization problem. We then study how this indirect welfare function responds to changes in the information set. This approach facilitates an analytical investigation of the interaction between equilibrium wages, private information, and redistributive policy.[9]

4.1 Full Information Ramsey Problem

It is well known that if the labor market is monopsonistic, a minimum wage can increase both employment and low-skilled wages, and therefore, improve efficiency (Card and Krueger, 1995; Manning, 2003; Robinson, 1933). Many papers have shown that the monopsony logic for the desirability of the minimum wage extends to other models of the labor market with frictions or informational asymmetries (Acemoglu, 2001; Cahuc et al., 2001; Drazen, 1986; Flinn, 2006; Jones, 1987; Lang, 1987; Rebitzer and Taylor, 1991; Swinnerton, 1996).[10] We focus on the

[9]Heathcote and Tsujiyama (2015) pursue a similar strategy to derive a closed-form expression for the social welfare function, although they focus on quantitatively assessing the optimality of the current U.S. income tax schedule and not its interaction with the minimum wage.

[10]There are other rationales for imposing a minimum wage. As argued by Allen (1987), the desirability of the minimum wage as a redistributive tool depends on the elasticity of substitution between the labor inputs in

case in which low-skilled workers are constrained by a minimum wage, while high-skill workers are unconstrained. We normalize $\theta_L = 1/\alpha$ without loss of generality.

Assumption 1. $\mu_L \approx 0$.

Assumption 1 is adopted to simplify the analytical exposition. How realistic is this assumption? In 2016, 79.9 million workers age 16 and older in the United States were paid at hourly rates, representing 58.7 percent of all wage and salary workers. Among those paid by the hour, 701,000 workers earned exactly the prevailing federal minimum wage of \$7.25 per hour. About 1.5 million had wages below the federal minimum. Together, these 2.2 million workers with wages at or below the federal minimum made up 2.7 percent of all hourly paid workers.[11]

In the Appendix, we derive closed-form solutions for competitive equilibrium price-allocation pairs (in terms of the policies τ and $\bar{w}$). This simplifies the analysis significantly. In particular, we can dispose of implementability constraints (eg. Atkinson and Stiglitz, 1980; Chari and Kehoe, 1999; Ljungqvist and Sargent, 2012), and write the Ramsey problem as the following *unconstrained* problem:

$$\max_{(\tau,\bar{w})\in[0,1]\times\mathbb{R}_+} \left\{ \begin{array}{ll} & \rho_L\left[\log\left(\bar{w}^{\alpha/(\alpha-1)}(1-\tau)+\tau\theta_H\alpha\left\{\frac{(1-\tau)}{\chi}\right\}^{\alpha}\right)-\chi\bar{w}^{1/(\alpha-1)}\right] \\ + & \rho_H\left[\log\left(\theta_H\alpha\left\{\frac{(1-\tau)}{\chi}\right\}^{\alpha}\right)-(1-\tau)\right] \end{array} \right\} \tag{SWF}$$

Consider first the tax decision. It is clear that the optimal tax is less than unity. We conjecture that the optimal income tax is positive and verify this later. The first order condition of the Ramsey problem w.r.t. τ is

$$\frac{\rho_L}{c_L}\left[-\bar{w}^{\alpha/(\alpha-1)}+\frac{\theta_H\alpha}{\chi^\alpha}\left\{(1-\tau)^\alpha-\tau\alpha(1-\tau)^{\alpha-1}\right\}\right]-\frac{\rho_H}{c_H}\left[\frac{\theta_H\alpha^2}{\chi^\alpha}(1-\tau)^{\alpha-1}\right]+\rho_H=0. \tag{2}$$

The optimal tax balances equity gains from redistribution with efficiency losses from distorting labor taxation. Now consider the minimum wage policy from the perspective of the Ramsey planner. The first order condition w.r.t. to $\bar{w}$ can be reduced to:

$$\alpha\bar{w}=\chi\bar{w}^{\alpha/(\alpha-1)}+\tau\theta_H\alpha\left\{\frac{(1-\tau)}{\chi}\right\}^{\alpha-1}. \tag{3}$$

the production technology. When the elasticity of substitution is infinite, and the labor market is competitive, then the minimum wage is not desirable as a redistributive tool on top of the optimal income tax. We abstract from these details as the purpose of the paper is not to justify the desirability of the minimum wage, but to characterize the relationship between changes in the optimal minimum wage and changes in optimal income taxation when both instruments are used.

[11] The percentage of hourly paid workers earning the prevailing federal minimum wage or less has been on a downward trend since 1979, which is when data began to be collected on a regular basis. It declined from 3.3 percent in 2015 to 2.7 percent in 2016 and remains well below the percentage of 13.4 recorded in 1979 (BLS).

Perfect substitution between the labor inputs of the production technology allows us to restrict the effects of perturbations to the minimum wage to just low-skilled workers. The optimal minimum wage is, therefore, independent of the Pareto weight on high-skilled individuals, and addresses just monopsony distortions.

4.1.1 Maximin Criterion

We focus on the determination of optimal public policy under the Maximin social welfare criterion.

Lemma 1. *Under the Maximin social welfare criterion, a unique interior solution for the optimal income tax exists. In particular, the optimal income tax is strictly positive and bounded above by* $\frac{1}{1+\alpha}$.

In this stylized setting, the upper-bound of income taxation is decreasing in α. Intuitively, societies with technologies that have a higher return to scale suffer larger efficiency losses per unit tax, and thus, induce lower optimal tax levels. Moreover, the optimal income tax is strictly positive. This verifies our initial guess.

We are now in a position to present the main results of this section. In particular, we vary the skill distribution and trace out implications on public policy. First, we deduce the effect of an increase in skill inequality on optimal public policies when they are independently determined. Equations (2) and (3) reveal that an increase in the productivity of high-skilled workers should always be accompanied by an increase in the income tax as well as an increase in the minimum wage when each policy is determined while keeping the other fixed. In particular, invoking the Implicit Function Theorem,

$$\begin{aligned}\frac{\partial \bar{w}}{\partial \theta_H}\Big|_{\tau \text{ fixed}} &= \frac{\tau\alpha\left\{\frac{(1-\tau)}{\chi}\right\}^{\alpha-1}}{\alpha+\chi[\alpha/(1-\alpha)]\bar{w}^{1/(\alpha-1)}} > 0, \text{ and} \\ \frac{\partial \tau}{\partial \theta_H}\Big|_{\bar{w} \text{ fixed}} &= \frac{\chi^{\alpha}\bar{w}^{\alpha/(\alpha-1)}}{(\theta_H\alpha)^2\{2(1-\tau)^{\alpha-1}+\tau(1-\alpha)(1-\tau)^{\alpha-2}\}} > 0.\end{aligned}$$

Evaluating the respective second order conditions under this paradigm, the proceeding lemma immediately follows.

Lemma 2. *Suppose that public policy is independently determined. Then under the Maximin social welfare criterion:*
(i) The optimal minimum wage is strictly increasing and linear in high-skilled productivity.
(ii) The optimal income tax is strictly increasing and concave in high-skilled productivity.

It is obvious, in this case, that the two redistributive instruments are complementary when responding to an increase in the skill gap. In contrast, it is less trivial to show that this positive relationship persists even when the minimum wage and the income tax are jointly determined. Under joint determination, for instance, the positive effect of the rising skill gap on the minimum wage can potentially be overturned by a simultaneous rise in the income tax. Nonetheless, the following lemma shows that even when the planner sets the minimum wage and the income tax in tandem, the increase in the utilization of one instrument is not enough to reduce the use of the other in tackling skill inequality.

Lemma 3. *In every Maximin Ramsey equilibrium, the optimal minimum wage and the optimal income tax are complements.*

4.2 The Incentive-constrained Planning Problem

Underreporting noncompliance is the largest component of the tax gap in the United States. The Internal Revenue Service (IRS) estimates that noncompliance from underreporting accounts for more than 80% of the tax gap. Furthermore, the Individual Income Tax is the single largest source of the annual tax gap, accounting for about two-thirds of the gap. Lastly, understated income, not overstated deductions, accounts for over 80% of individual underreporting (Tax Gap Facts and Figures, IRS).[12] Motivated by these facts, we consider a setting in which individuals privately observe their productivity. In this setting, we show that the government may find it optimal to set a public policy in which the minimum wage and the income tax share a negative relationship to discourage the misreporting of information.

The government can access an auditing technology Φ which is dependent on the policy instruments. Due to the Revelation Principle, the value of truthfully revealing your type must be larger than the value of deviating. When mimicking low-skilled workers, high-skilled workers will produce $y_L = \bar{w}^{\alpha/(\alpha-1)}/\alpha$ units of output. Producing this level of output requires $\tilde{l}_H = \bar{w}^{1/(\alpha-1)}/(\alpha\theta_H)^{1/\alpha}$ units of work effort. Moreover, in the process of mimicking low-skilled workers, high-skilled workers supply labor at the minimum wage. Thus, all Pareto efficient allocations must satisfy the following incentive constraint: $\log(c_H) - \chi l_H \geq \log(c_L) - \chi\tilde{l}_H + \Phi(\tau, \bar{w})$. Notice that when $\Phi(.) = 0$, this reduces to the standard incentive compatibility constraint under labor income risk without costly state verification (Townsend, 1979; Gale and Hellwig, 1985). For tractability, we restrict attention to the Maximin social welfare function and consider cases in which the incentive constraint for low-skilled workers is slack.[13]

Assumption 2. $\Phi : [0,1] \times \mathbb{R}_+ \mapsto \mathbb{R}$ *is of class* $\mathcal{C}^2$ *such that* $\Phi_{ii} \geq 0 \ \forall i \in \{1, 2\}$.

[12] The IRS comes up with its estimates by combining information from a program of random intensive audits, originally known as the Taxpayer Compliance Measurement Program (TCMP). The estimates we report are from the a modified version of the TCMP, called the National Research Program (NRP), which was implemented to examine individual income tax returns from the 2001 tax year.

[13] The latter can be guaranteed if the range of the auditing technology is contained in the negative orthant of the Euclidean space.

Assumption 2 restricts the auditing technology to twice continuously differentiable and (weakly) convex functions. As shown by the following lemma, this guarantees that the first order conditions are sufficient for isolating global maxima.

Lemma 4. *Under assumption 2, the Maximin social welfare function is (strictly) concave.*

The incentive-constrained Maximin Ramsey problem is given by

$$\max_{(\tau,\bar{w})\in[0,1]\times\mathbb{R}_+} \quad \log\big(\bar{w}^{\alpha/(\alpha-1)}(1-\tau)+T\big)-\chi\bar{w}^{1/(\alpha-1)}$$

$$\text{s.t.} \quad \log\big(T/\tau\big)-(1-\tau)\geq \log\big(\bar{w}^{\alpha/(\alpha-1)}(1-\tau)+T\big)-\chi\frac{\bar{w}^{1/(\alpha-1)}}{(\alpha\theta_H)^{1/\alpha}}+\Phi(\tau,\bar{w})$$

$$T=\tau\theta_H\alpha\left\{\frac{(1-\tau)}{\chi}\right\}^{\alpha}$$

For illustrative purposes, we first investigate the relationship between the minimum wage and the income tax in partial equilibrium. That is, keeping skill levels constant, we trace out how one policy instrument changes when the other is exogenously raised. Let the Lagrange multiplier on the incentive constraint be denoted by $\lambda \geq 0$. The first order condition w.r.t. to τ is

$$(1-\lambda)\frac{\bar{w}^{\alpha/(\alpha-1)}}{c_L}=(1-\lambda)\frac{\theta_H\alpha}{c_L\chi^{\alpha}}(1-\tau)^{\alpha-1}\big\{1-\tau(1+\alpha)\big\}-\lambda\left\{\frac{\theta_H\alpha^2}{c_H\chi^{\alpha}}(1-\tau)^{\alpha-1}+\Phi_1(\tau,\bar{w})-1\right\}.$$

Notice that when $\lambda > 0$, then $\frac{\partial\tau}{\partial\bar{w}} \lesseqgtr 0$. When $\lambda = 0$, we get back (4) which implies $\frac{\partial\tau}{\partial\bar{w}} > 0$. Furthermore, the first order condition w.r.t. $\bar{w}$ can be reduced to

$$\frac{1-\lambda}{c_L}\big[\alpha\bar{w}(1-\tau)\big]=\chi\left[1-\frac{\lambda}{(\alpha\theta_H)^{1/\alpha}}\right]+\lambda\Phi_2(\tau,\bar{w}).$$

Observe that the above equation again implies that when $\lambda > 0$, then $\frac{\partial\tau}{\partial\bar{w}} \lesseqgtr 0$. That is, in the incentive-constrained problem, the relationship between changes in the minimum wage and changes in the income tax is ambiguous in general.

Remark. *The optimal minimum wage and the optimal income tax can be substitutes when workers privately observe their productivity.*

The next lemma shows that substitution between the two policy instruments is more likely when skill dispersion is relatively low.

Lemma 5. *If* Φ *is bounded, then* $\lim_{\theta_H\to\infty}\lambda = 0$.

Figure 5 captures how the relationship between the optimal minimum wage and the optimal income tax under full information ($\lambda = 0$) differs from the asymmetric information case ($\lambda > 0$).

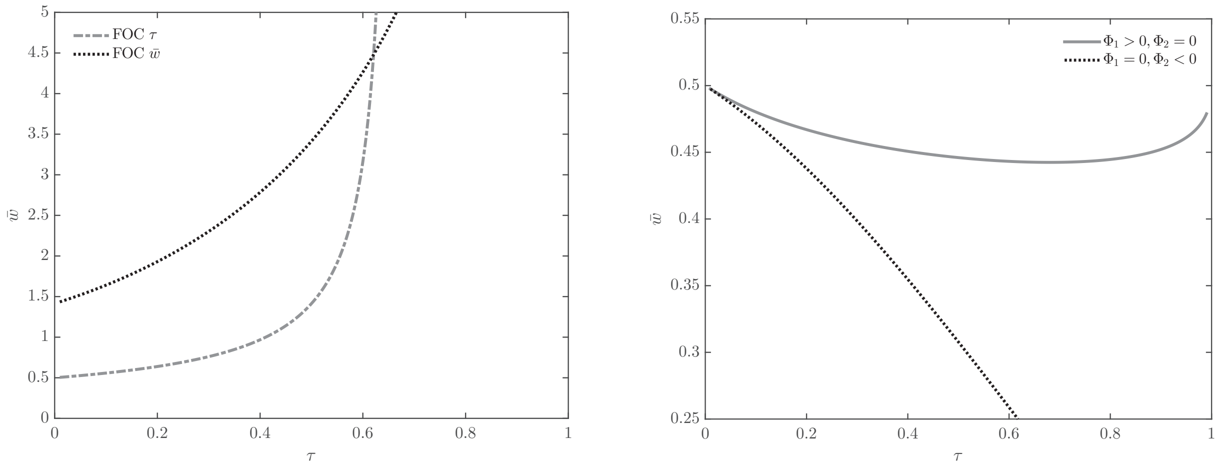
Plot (a) depicts the optimality conditions under the full information economy. It shows that the partial equilibrium relationship between the two redistributive tools is strictly monotonic in the full information economy. Plot (b) shows that this relationship breaks down under asymmetric information. Though we allow the auditing technology to be dependent on both policy instruments, the positive relationship between the minimum wage and the income tax breaks down even when we restrict the auditing technology to respond only to changes in the minimum wage or the income tax in isolation. This is illustrated in the following two cases.

We focus on cases in which either $\Phi_1 > 0$ or $\Phi_2 < 0$, both of which capture the idea that the higher the income of the individual the lower the incentive to misreport their type.[14] The empirical literature does not provide estimates for the elasticity of the auditing technology with respect to income. Though the IRS does not explicitly mention why auditors single out some returns for extra scrutiny, we can get a sense of which groups draw most attention of the IRS from their track record. Table 4 shows that individuals with higher incomes are more likely to be audited by the IRS. About 14.5 percent of individuals earning in the seven digits were audited, far above the overall rate of 0.62 percent (IRS Data Book 2017). The empirical literature on tax evasion also provides some support for this claim. Christian (1994) finds, based on the 1988 TCMP study, that higher-income people evade less than those with lower incomes, relative to the size of their true income. According to this study those with adjusted gross income above \$500,000 on average reported 97.1 percent of their true incomes to the IRS, compared to just 78.7 percent for those with adjusted gross income between \$5,000 and \$10,000. A similar finding emerges from studies on tax evasion in big business. The IRS estimates that the noncompliance rate of larger companies is lower: 14 percent, compared to 29 percent for corporations with less than \$10 million of assets.

First, consider the case in which $\Phi_1 > 0$ and $\Phi_2 = 0$, i.e. increasing the income tax reduces the effectiveness of the auditing technology on the margin. As pointed out by Slemrod (2017), the tax base can have a profound effect on the enforceability of taxation, and the enforcement regime can affect the impact of tax rates on taxpayer decisions that determine the tax base. In this scenario, the non-monotonic relationship between the policy instruments stems from the counteracting effects that increasing the income tax has on the value of deviation. It reduces the consumption allocation of the low-skilled worker (which decreases the value of the deviation) and reduces the efficacy of income monitoring (which increases the value of the deviation).

Consider now the case in which $\Phi_1 = 0$ and $\Phi_2 < 0$, i.e. increasing the minimum wage facilitates income monitoring. The relationship between the policy instruments is, in general, non-monotonic in this case as well. In this scenario, the positive relationship between the policy

[14] We hone in on the main mechanism by considering cases in which $\Phi_{ij} = 0 \; \forall (i,j) \in \{1,2\},\; i \neq j$. This shuts down any substitution between the inputs of the auditing technology itself, and ensures that the non-monotonicity between the policy instruments emerges from the interaction between the auditing technology and the incentives to misreport productivity.



(a) Full Information (b) Asymmetric Information

Notes: The first plots displays the optimality conditions for the income tax and the minimum wage under full information ($\lambda = 0$). The second plot displays the relationship between the policy instruments under a binding incentive constraint. The parameterization is as follows: $(\alpha, \theta_H, \chi) = (0.5, 2/\alpha, 1)$. For the asymmetric information plot, the case in which $\Phi_1 > 0$ and $\Phi_2 = 0$ corresponds to $\Phi(\tau) = \tau$, and the case in which $\Phi_1 = 0$ and $\Phi_2 < 0$ corresponds to $\Phi(\bar{w}) = \tilde{l}_H - 1$.

Figure 5: Partial Equilibrium Relationship b/w Minimum Wage and Income Tax

instruments breaks down due to the opposing effects that the minimum wage and the income tax have on the incentives for truthful revelation. Increasing the income tax increases the relative value of the deviation due to redistributive motives. On the other hand, raising the minimum wage has two effects on the value of deviation. First, low-skilled workers receive higher wages, which increases the value of the deviation. Second, the auditing technology becomes more effective, which decreases the value of the deviation. If the latter effect dominates, then the relationship between the two policy instruments can be non-monotonic.

We have already established that changes in the minimum wage and changes in the income tax go hand in hand when incentive constraints are slack. We now isolate a condition on the government's auditing technology that is sufficient to guarantee that the complementary relationship between the two instruments holds even when incentive constraints bind. Under binding incentive constraints, the sufficient condition for complementarity turns out to be dependent on the cross partial derivative of the auditing technology.

Assumption 3. $\exists c \in \mathbb{R}_+ \; \exists \tilde{\Phi} : [0,1] \times \mathbb{R}_+ \mapsto \mathbb{R}$ *independent of* $c : \Phi(.) = \tilde{\Phi}(.) + c$.

Lemma 6. *Suppose assumption 3 holds,* $\Phi_{12} < 0$ *and* c *is large enough. Then the following conditions are met when workers privately observe their productivity:*
(i) The Maximin social welfare function is (strictly) supermodular in policy variables $(\tau, \bar{w})$.
(ii) The Maximin social welfare function exhibits increasing differences in $(\tau, \bar{w}; \theta_H)$.
(iii) The optimal minimum wage and the optimal income tax are complements.

4.2.1 Strategic Complementarity in Public Policy

We now consider the case in which the minimum wage and the income tax are strategically determined. In this setting, we are interested in conditions under which the minimum wage and the income tax are strategic complements. This class of games was introduced by Topkis (1979), and further developed by Milgrom and Roberts (1990) and Vives (1990). The main result of this section is that the sufficient condition for complementarity on the cross partial derivative of the auditing technology that we isolated earlier is robust to environments in which public policy is determined strategically.

Observe that it will never be optimal to set a wage floor above w_H. This ensures that the choice set for the minimum wage is compact. Define

$$B_\tau(\bar{w}) = \text{argmax}_{\tau\in[0,1]}\text{SWF}, \text{ and } B_{\bar{w}}(\tau) = \text{argmax}_{\bar{w}\in[0,w_H]}\text{SWF}.$$

Define a map $B : [0,1] \times [0, w_H] \mapsto [0,1] \times [0, w_H]$ as

$$B(\tau, \bar{w}) = (B_\tau(\bar{w}), B_{\bar{w}}(\tau)).$$

Define a game $\Gamma = (2, \{A_\tau, A_{\bar{w}}\}, \{\text{SWF}, \text{SWF}\})$, where 2 represents the number of players, $A_\tau : [0, w_H] \mapsto [0,1]$ and $A_{\bar{w}} : [0,1] \mapsto [0, w_H]$ denotes the action sets, and SWF captures the payoff functions. A fixed point of B is a Nash Equilibrium of Γ.

Definition 2. *The minimum wage and the income tax are strategic complements if B is coordinatewise increasing, i.e.*

$$B_\tau(\bar{w}') > B_\tau(\bar{w}) \ \forall \bar{w}' > \bar{w}, \textit{ and } B_{\bar{w}}(\tau') > B_{\bar{w}}(\tau) \ \forall \tau' > \tau.$$

Lemma 7. *Suppose $\Phi_{12} < 0$. Then a unique pure-stratergy Nash Equilibrium of Γ exists in which the minimum wage and the income tax are strategic complements.*

5 Quantitative Analysis

We discipline the model using U.S. data over the same sample period that we used to obtain the empirical estimates presented earlier: 2000–2015. This period saw a persistent rise in the skill premium. The increase in earnings inequality in the U.S. is not unique to this period. Its steady rise since 1953 has been well documented by Kopczuk et al. (2010) using Social Security Administration longitudinal data. Acemoglu et al. (2001) argue that a SBTC is at the root of the rise in inequality. We find that the substantial rise in the traditional notion of the skill premium, which compares wages of college to non-college workers, are mimicked by the MWASP to a large extent. In this section, we map this exogenous increase in the MWASP to changes in public policy. Then, we estimate the auditing technology (in each state) by matching the simulated correlation between the minimum wage and the income tax with its data counterpart. Public policy is determined jointly in all of the exercises presented in this section.

5.1 Calibration

Labor Share. The labor share, α, is calibrated so that the average tax level in the model (across time) matches the average tax level (across location-time) in the data of 58%.[15] This yields an estimate of $\alpha = 0.7$, which is broadly consistent with the labor share in the United States.[16]

Labor Disutility. The labor disutility coefficient, χ, is calibrated so that the average simulated wage in the asymmetric information case coincides with the average realized federal minimum wage. This yields an estimate of 3.25.

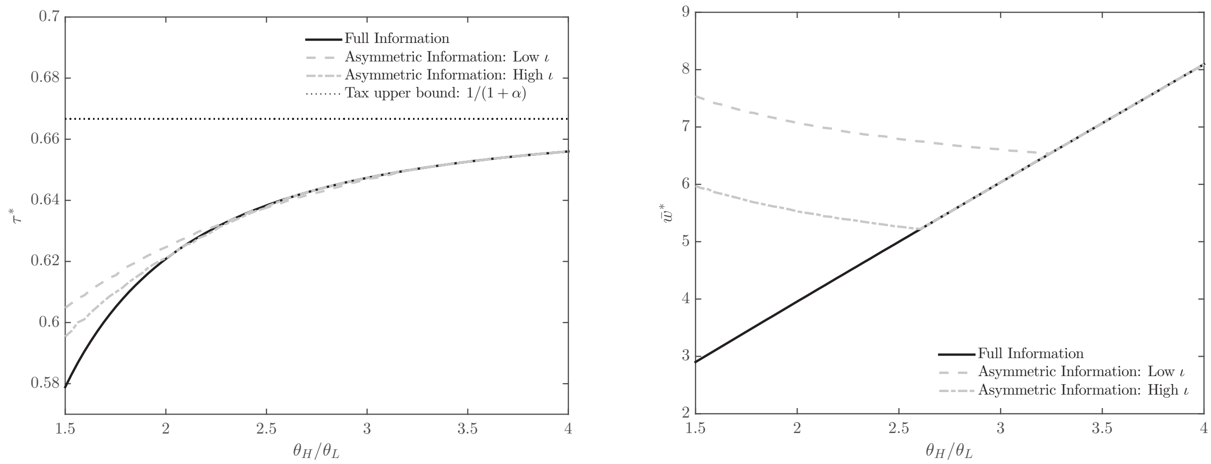
Auditing Technology. We restrict attention to the case in which $\Phi_1 = 0$ and $\Phi_2 < 0$. In particular, we assume that the governments auditing technology can be represented by the following functional form: $\Phi(\tau, \bar{w}) = \bar{w}^{-\iota}$, where $\iota > 0$. The parameter ι is crucial in determining the extent to which incentive constraints bind. This is calibrated so that the minimum wage in pre-crisis years remains unchanged at 5.15 USD per hour to be consistent with the data. This yields an estimate of $\iota = 1.4$.

Skill Premium Shocks. Skill premium shocks are fed from the data as per the following procedure. Relying on CPS data, the labor literature has mainly focused on returns to college education (Acemoglu and Autor, 2011). This may not be an appropriate measure of the skill premium for the purposes of our model. A suitable indicator should instead compare a measure of average skill (college and non-college, barring minimum wage employees) to a measure of skills of minimum wage workers. To construct a measure for the former, we use

$$w_{Ht} = \frac{w_t - \mu_{Lt}\bar{w}_t}{1 - \mu_{Lt}},$$

[15]The overall tax burden is measured by Federal Tax Rate for Wages + State Tax Rate for Wages + Federal Tax Rate for Long Gains + State Tax Rate for Long Gains + State Tax Rate for Mortgage Interest (deduction), where we remove variation from changes in federal rates to focus attention on variation at the state-level.

[16]The BLS estimates that labor share has declined from about 0.7 to about 0.5 during the second half of the 20th century. In sections 5.2 and 5.4, we employ $\alpha = 0.5$. In this alternative parametrization, the labor disutility coefficient, χ, is re-calibrated so that the average full information minimum wage across the simulated time period equals the average federal minimum wage over the sample period. This yields $\chi = 0.8$.



(a) Income Tax

(b) Minimum Wage

Figure 6: Policy Functions

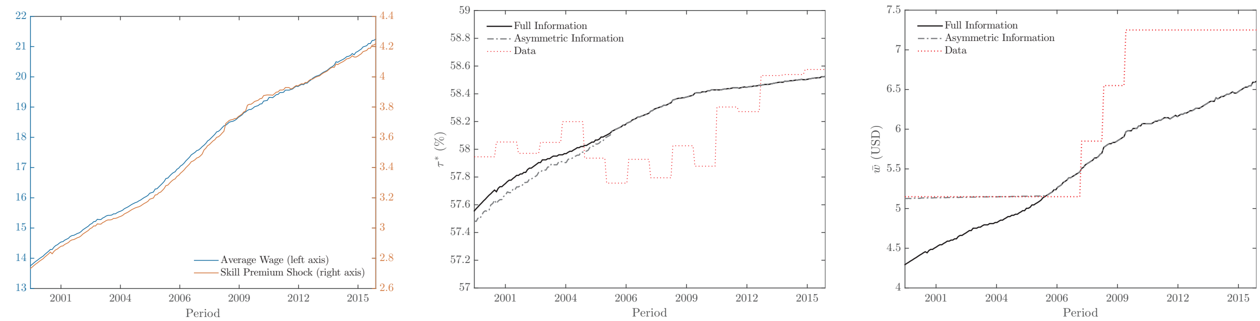
where w_t denotes the average hourly earnings of all employees (total private) in period t (FRED), and μ_{Lt} denotes the percentage of hourly paid workers earning at or below prevailing federal minimum wage in period t (BLS). Then, the skill premium (relative to minimum wages) is imputed using the ratio $w_{Ht}/\text{Mo}(\bar{w}_t)$.

5.2 Policy Functions

Figure 6 depicts how the optimal minimum wage and the optimal income tax vary when the skill distribution becomes more dispersed. Notice that in response to an increase in skill inequality, the planner increases the utilization of both redistributive instruments in tandem to combat income inequality when it can perfectly observe worker productivity levels. Under asymmetric information, however, the optimal minimum wage and the optimal income tax can be substitutes over low levels of high-skilled productivity. As is the case in the full information economy, the optimal tax level increases as worker skills diverge. At the same time, the planner finds it optimal to reduce the minimum wage to discourage high-skilled workers from misreporting their type. Nonetheless, when skill dispersion is sufficiently high, the value of this deviation is outweighed by the gain from truthful reporting, and the full information allocation is recovered; see Lemma 5.

The figure also illustrates how changes in the auditing technology can affect public policy. An increase in the responsiveness of the auditing technology to changes in the minimum wage, captured by $1/\iota$, increases the probability of the minimum wage and the income tax being negatively correlated.

Lastly, irrespective of the level of skill dispersion, the optimal minimum wage is always (weakly) greater under asymmetric information compared to the benchmark. In contrast, the



(a) Realized Shocks (b) Optimal Income Tax (c) Optimal Minimum Wage

Figure 7: Changes in Optimal Public Policy in Response to Realized Variations in Skill Dispersion over 2000-2015

difference between the optimal tax under asymmetric information and the optimal tax under full information is positive over low levels of skill dispersion, negative over medium levels of skill dispersion, and zero over high levels of skill dispersion.

5.3 Response to Observed Skill-biased Technical Change

Figure 7 depicts the optimal response of public policy to realized changes in skill dispersion in the U.S. over the period 2000–2015. Figure 7(a) plots the observed average hourly earnings of all employees with our constructed measure of the skill premium. By feeding the implied skill dispersion shocks into the model, we simulate the optimal income tax and the optimal minimum wage under full information as well as asymmetric information. Figures 7(b)–(c) plot the simulated estimates of the optimal income tax and the optimal minimum wage respectively, against their data counterparts. These figures reveal that the model tracks the data closely. These estimates suggest that asymmetric information constrained the optimal public policy in the pre-crisis period when the skill premium was relatively low.

Table 3 shows that the simulations from the asymmetric information model match the calibration targets. The table also shows that the model does a good job in mimicking non-targeted moments. We find that the full information model can account for 87% of the variation in the minimum wage and 26% of the variation in taxation, while the asymmetric information model can account for 94% of the variation in the minimum wage and 23% of the variation in taxation.[17] In the full information model, the minimum wage and the income tax always share a perfect positive correlation; see Lemma 3. As a result of this property, an increase in the skill premium is entirely passed through to an increase in the utilization of both policy instruments. However, in the pre-crisis period (2000-2006), the federal minimum wage remained constant at 5.15 USD per hour while the skill premium increased by about 25 percent. The asymmetric

[17]For each estimate, we use R-squared from an OLS regression of the observed time-series from the data on the simulated time-series from the corresponding model with a constant term.

information model, in contrast, can rationalize this narrative.

Table 3: Summary Statistics

Moments	Model 1	Model 2	Data
Mean τ^*			
Pre-crisis$^\diamond$	57.9	57.9	57.9
Full sample	58.2	58.2	58.1
Mean $\bar{w}^*$			
Pre-crisis$^\diamond$	4.76	5.15	5.15
Full sample	5.48	5.65	6.07
$\sigma(\tau^*)$			
Pre-crisis	0.17	0.20	0.13
Full sample	0.28	0.31	0.26
$\sigma(\bar{w}^*)$			
Pre-crisis	0.26	0.02	0
Full sample	0.71	0.52	0.98

Notes: Model 1 corresponds to the full information setting, while Model 2 corresponds to the asymmetric information setting. The pre-crisis period ranges from 2000–2006, and the full sample ranges from 2000–2015. $^\diamond$ denotes targeted moments in Model 2. Tax rates (τ^*) are measured in percentages. Wage floors ($\bar{w}^*$) are measured in USD per hour.

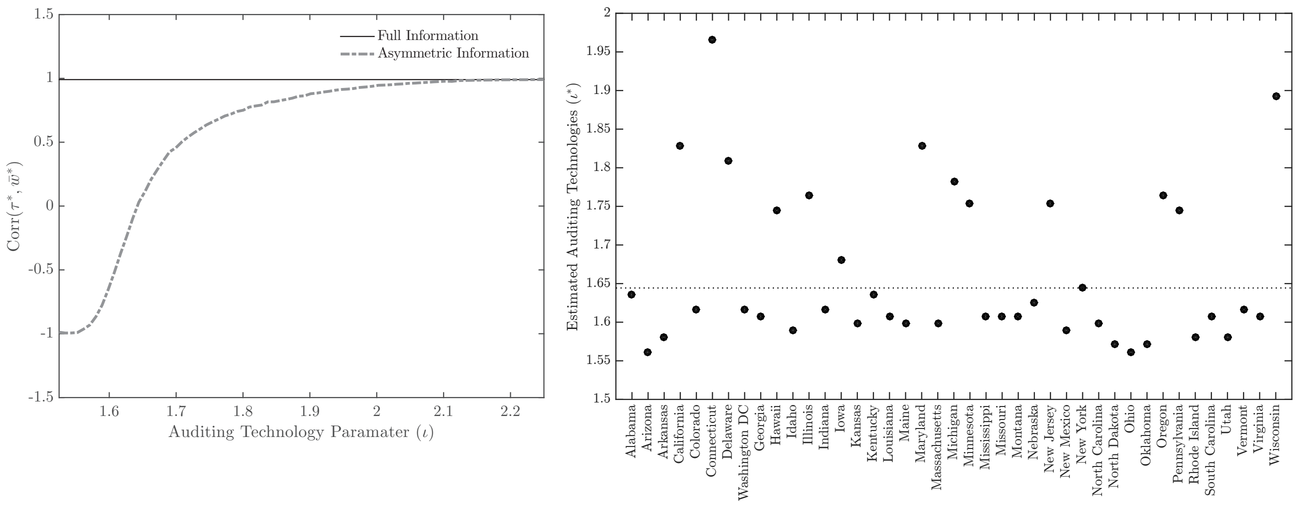
5.4 Estimating Auditing Technologies

We now vary the auditing technology parameter, ι, and map changes to the correlation between the simulated policy variables. For this exercise, we consider a linear increase in skill dispersion (θ_H/θ_L) from 2.5 to 3.5. This is consistent with fact that the average hourly earnings of all employees increased steadily from about 14 USD to about 21 USD (BLS), while the average federal minimum wage over 2000-2015 was about 6 USD. We find that the correlation between the optimal minimum wage and the optimal income tax is decreasing in the sensitivity of the auditing technology; see Figure 8(a). For low values of ι the correlation is close to -1, while for high values of ι the correlation is close to 1. Since Corr$(\tau^*, \bar{w}^*)$ is a smooth function of ι under the baseline calibration, we can back-out the effectiveness of audits that would be required in each state to deliver the correlations (for levels) that we observe in the data that were presented in section 2. In particular, we estimate the 41x1 vector of auditing technologies using simulated method of moments (SMoM):

$$\iota^* \in \text{argmin}_{\iota \in \mathbb{R}_+} \mathcal{D}(\iota)' W \mathcal{D}(\iota),$$

where $\mathcal{D}(\iota) \equiv \text{Corr}(\tau^*_{\text{Model}}, \bar{w}^*_{\text{Model}} \mid \iota) - \text{Corr}(\tau^*_{\text{Data}}, \bar{w}^*_{\text{Data}})$, and W is 41x41 weighting matrix.[18] Figure 8(b) reports the estimates when W is set to the identity matrix.

[18]We only report estimates for 41 states since data in only these states exhibit variation.



(a) Model Correlations

(b) SMoM Estimates of Auditing Technology

Notes: The higher the parameter (ι), the less sensitive is the auditing technology to changes in the minimum wage. The dotted line in plot (b) represents the technology under which the correlation between the minimum wage and the income tax is zero.

Figure 8: Estimating Auditing Technologies Across U.S. States

6 Conclusion

This paper studied the interaction between two commonly used redistributive tools: the minimum wage and the income tax. Empirically, we document that the minimum wage and the income tax are negatively correlated in a majority of U.S. states over the period 2000-2015, and find that the hypothesis of complementarity between the two redistributive instruments is rejected in the data. Moreover, we find that a SBTC can explain much of the variation in these policy instruments. Theoretically, we show that a frictionless model cannot rationalize these findings. In particular, using the Maximin criterion for social welfare, we find that the minimum wage and the income tax are complements in combating skill inequality under complete information. When workers privately observe their productivity, in contrast, the two instruments can be substitutes under realistic restrictions on the government's auditing technology. Quantitatively, we illustrate how differences in the sensitivity of the auditing technology to changes in income, a feature corroborated by the literature on tax evasion and the IRS' track record, can account for the observed cross-sectional correlations between the minimum wage and the income tax.

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Appendix

Proof of Lemma 1: We conjecture that the solution is interior and verify this later. When $\rho_H = 0$, equation (2) reduces to

$$\bar{w}^{\alpha/(\alpha-1)} = \frac{\theta_H \alpha}{\chi^\alpha}(1-\tau)^{\alpha-1}\big\{1-\tau(1+\alpha)\big\}. \tag{4}$$

Observe that condition (4) imposes an endogenous upper-bound on linear income taxes. In particular, since the minimum wage is non-negative and income taxes are strictly less that unity, it must be that

$$\tau \leq \frac{1}{1+\alpha}.$$

We are interested in $(\tau, \bar{w})$ that solve the 2x2 system comprised of equations (3) and (4). Notice that equation (4) admits a closed-form solution for $\bar{w}$, which we can substitute into (3) to arrive at one equation in τ:

$$\alpha\big\{1-\tau(1+\alpha)\big\} = \theta_H^{1/(\alpha-1)}(1-\tau)(1-\tau\alpha)^{\alpha/(\alpha-1)}.$$

Notice that since $1-\tau(1+\alpha)$ is non-negative, it must be that $1-\tau\alpha$ is strictly positive. Let

$$\mathcal{F}(\tau, \theta_H) \equiv \theta_H^{1/(\alpha-1)}(1-\tau)(1-\tau\alpha)^{\alpha/(\alpha-1)} - \alpha\big\{1-\tau(1+\alpha)\big\}.$$

Consider the boundary conditions:

$$\begin{aligned}\mathcal{F}(0, \theta_H) &= \theta_H^{1/(\alpha-1)} - \alpha < 0,\\ \mathcal{F}(1, \theta_H) &= \alpha^2 > 0.\end{aligned}$$

One can prove the first inequality by contradiction. Suppose $\mathcal{F}(0, \theta_H) \geq 0$. This implies $\theta_L \geq \theta_H^{1/(1-\alpha)} > \theta_H$. But this cannot be true as $\theta_H > \theta_L$ by assumption. Furthermore, $\mathcal{F}$ is a composition of polynomials and, therefore, continuous everywhere. Thus, by the intermediate value theorem $\exists \tau^* \in (0,1) : \mathcal{F}(\tau^*, \theta_H) = 0$. Uniqueness follows as (SWF) is (strictly) concave. $\square$

Proof of Lemma 3: We first show that $\frac{\partial \tau}{\partial \theta_H} > 0$. To see this, notice that $\mathcal{F}^a(\tau) \equiv \alpha\{1-\tau(1+\alpha)\}$ is strictly decreasing in τ. Moreover, $\mathcal{F}^b(\tau, \theta_H) \equiv \theta_H^{1/(\alpha-1)}(1-\tau)(1-\tau\alpha)^{\alpha/(\alpha-1)}$ is strictly decreasing in τ. An increase in θ_H shifts $\mathcal{F}^b(\tau, \theta_H)$ downward and leaves $\mathcal{F}^a(\tau)$ unchanged. Since the optimal income tax must satisfy $\mathcal{F}^a(\tau) = \mathcal{F}^b(\tau, \theta_H)$, τ must also be strictly increasing in θ_H. Figure 9 graphically illustrates this argument.

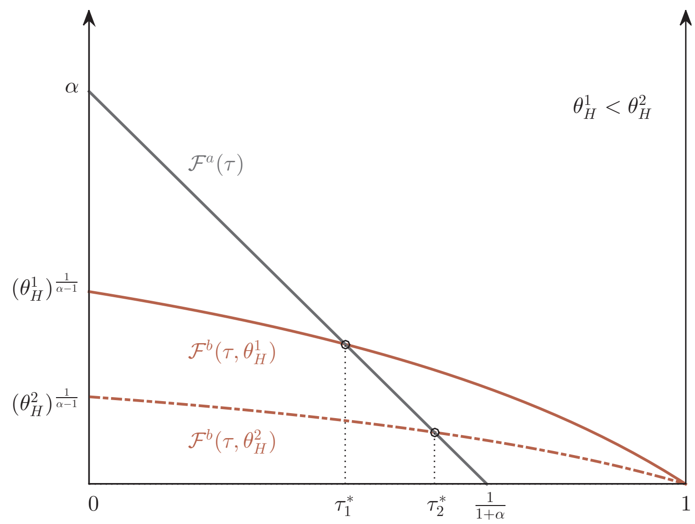

Figure 9: Deducing the Relationship b/w the Optimal Tax and Productivity Shocks

Let $\mathcal{F}^{\bar{w}}(\tau, \bar{w}, \theta_H) \equiv \alpha\bar{w} - \chi\bar{w}^{\alpha/(\alpha-1)} - \tau\theta_H\alpha\{\frac{(1-\tau)}{\chi}\}^{\alpha-1}$. A necessary condition for the optimal minimum wage policy is $\mathcal{F}^{\bar{w}}(\tau, \bar{w}, \theta_H) = 0$. Totally differentiating this expression,

$$\frac{\partial \bar{w}}{\partial \theta_H} = -\left[\underbrace{\frac{\partial \mathcal{F}^{\bar{w}}}{\partial \tau}}_{<0} \times \underbrace{\frac{\partial \tau}{\partial \theta_H}}_{>0} + \underbrace{\frac{\partial \mathcal{F}^{\bar{w}}}{\partial \theta_H}}_{<0}\right] / \underbrace{\frac{\partial \mathcal{F}^{\bar{w}}}{\partial \bar{w}}}_{>0} > 0.$$

Thus, $sgn\left(\frac{\partial \tau}{\partial \theta_H} \times \frac{\partial \bar{w}}{\partial \theta_H}\right) = 1.$ $\square$

Proof of Lemma 4: First, we consider the unconstrained case ($\lambda = 0$). Here, SWF $|_{\lambda=0}$ $= \log\left(\bar{w}^{\alpha/(\alpha-1)}(1-\tau) + \tau\theta_H\alpha\{\frac{(1-\tau)}{\chi}\}^{\alpha}\right) - \chi\bar{w}^{1/(\alpha-1)}$. Thus,

$$\frac{\partial^2 \text{SWF}}{\partial \tau^2} |_{\lambda=0} = -\frac{1}{c_L^2}\left\{\frac{\theta_H\alpha}{\chi^\alpha}(1-\tau)^{\alpha-1}\{1-\tau(1+\alpha)\} - \bar{w}^{\alpha/(\alpha-1)}\right\}^2 + \frac{\theta_H\alpha^2}{\chi^\alpha c_L}(1-\tau)^{\alpha-2}\left\{\tau(1+\alpha) - 2\right\}.$$

Notice that since $\alpha < 1$, $\tau(1+\alpha) - 2 < 0$. Thus, $\frac{\partial^2 \text{SWF}}{\partial \tau^2} < 0$. Moreover,

$$\frac{\partial^2 \text{SWF}}{\partial \bar{w}^2} |_{\lambda=0} = -\chi[1/(\alpha-1)][1/(\alpha-1)-1]\bar{w}^{1/(\alpha-1)-2} - \frac{1}{c_L^2}\left\{[\alpha/(\alpha-1)]\bar{w}^{\alpha/(\alpha-1)-1}(1-\tau)\right\}^2$$
$$+\frac{1}{c_L}\left\{[\alpha/(\alpha-1)][\alpha/(\alpha-1)-1]\bar{w}^{\alpha/(\alpha-1)-2}(1-\tau)\right\}.$$

Again, since $[1/(\alpha-1)] < 0$, observe that $\frac{\partial^2 \text{SWF}}{\partial \bar{w}^2} |_{\lambda=0} < 0$.

Next, we confront the constrained case ($\lambda > 0$). Substituting the binding incentive constraint into the objective function:

$$\text{SWF} \mid_{\lambda>0} = \log\left(\theta_H \alpha \left\{\frac{(1-\tau)}{\chi}\right\}^{\alpha}\right) - (1-\tau) + \chi\frac{\bar{w}^{1/(\alpha-1)}}{(\alpha\theta_H)^{1/\alpha}} - \Phi(\tau,\bar{w}) - \chi\bar{w}^{1/(\alpha-1)}.$$

As Φ twice continuously differentiable, partially differentiating the expression w.r.t. τ:

$$\frac{\partial^2 \text{SWF}}{\partial \tau^2} \mid_{\lambda>0} = -\frac{1}{c_H^2}\left\{-\frac{\theta_H \alpha^2}{\chi^{\alpha}}(1-\tau)^{\alpha-1}\right\}^2 - \frac{1}{c_H}\left\{-\frac{\theta_H\alpha^2(\alpha-1)}{\chi^{\alpha}}(1-\tau)^{\alpha-2}\right\} - \Phi_{11}(\tau,\bar{w}).$$

Notice that if $\Phi_{11}(\tau,\bar{w}) \geq 0$, then $\frac{\partial^2 \text{SWF}}{\partial \tau^2} < 0$. Moreover,

$$\frac{\partial^2 \text{SWF}}{\partial \bar{w}^2} \mid_{\lambda>0} = \chi[1/(\alpha\theta_H)^{(1/\alpha)} - 1][1/(\alpha-1)][1/(\alpha-1)-1]\bar{w}^{1/(\alpha-1)-2} - \Phi_{22}(\tau,\bar{w}).$$

Since $\theta_H > 1/\alpha$ and $\alpha < 1$, we have $\frac{\partial^2 \text{SWF}}{\partial \bar{w}^2} \mid_{\lambda>0} < 0$ when $\Phi_{22}(\tau,\bar{w}) \geq 0$. Thus, the (weak) convexity of the government's auditing technology (assumption 2) is sufficient to guarantee that the first order conditions are sufficient in isolating global maxima. □

Proof of Lemma 5: A direct consequence of Lemma 6(i)-(ii) is that $\frac{\partial \bar{w}}{\partial \theta_H} > 0$. This implies that

$$\lim_{\theta_H\to\infty} \chi\frac{\bar{w}^{1/(\alpha-1)}}{(\alpha\theta_H)^{1/\alpha}} = 0,$$

and

$$\lim_{\theta_H\to\infty} \log(\bar{w}^{\alpha/(\alpha-1)}(1-\tau) + T) < \infty.$$

Since Φ is bounded, these conditions are sufficient to guarantee that the value of deviation is also bounded in the limit. We are left to show that the value of truthful reports tends to infinity as high-skilled productivity tends to infinity. We show this using guess and verify. In particular, we conjecture that the incentive constraint is slack and, hence, Lemma 1 applies. In particular, τ is strictly positive and bounded above by $1/(1+\alpha) < 1$, which necessitates

$$\lim_{\theta_H\to\infty} \log\left(\theta_H \alpha \left\{\frac{(1-\tau)}{\chi}\right\}^{\alpha}\right) = \infty.$$

The above condition holds as, in the limit, the direct effect of an increase in the consumption of high-skilled workers due to an increase in wages and hours worked outweighs the decrease in the consumption of high-skilled workers that results due to a compensating increase in income taxation. This verifies our guess that incentive constraints are slack in the limit. □

Proof of Lemma 6: If c is large enough, then the incentive constraint binds. In this case, we can re-write this constraint as:

$$\log\left(\bar{w}^{\alpha/(\alpha-1)}(1-\tau)+T\right)=\log\left(\theta_H\alpha\left\{\frac{(1-\tau)}{\chi}\right\}^{\alpha}\right)-(1-\tau)+\chi\frac{\bar{w}^{1/(\alpha-1)}}{(\alpha\theta_H)^{1/\alpha}}-\Phi(\tau,\bar{w}).$$

Substituting this into the objective function we can reduce the maximization problem to:

$$\max_{(\tau,\bar{w})\in[0,1]\times\mathbb{R}_+}\qquad \text{SWF}\equiv\log\left(\theta_H\alpha\left\{\frac{(1-\tau)}{\chi}\right\}^{\alpha}\right)-(1-\tau)+\chi\frac{\bar{w}^{1/(\alpha-1)}}{(\alpha\theta_H)^{1/\alpha}}-\Phi(\tau,\bar{w})-\chi\bar{w}^{1/(\alpha-1)}$$

Notice that SWF is twice continuously differentiable on $[0,1]\times\mathbb{R}_+\times\mathbb{R}_+\setminus[0,1/\alpha)$. Thus, by a theorem from Topkis (1978), a sufficient condition for the (strict) supermodularity of SWF is $\frac{\partial^2\text{SWF}}{\partial i\partial j}>0\ \forall i\neq j,\ i,j\in\{\tau,\bar{w}\}$. Observe that under the restriction on the government's auditing technology delineated in the statement of the lemma, we have

$$\frac{\partial^2\text{SWF}}{\partial\tau\partial\bar{w}}=-\Phi_{12}(\tau,\bar{w})>0.$$

This establishes that SWF is supermodular in $(\tau,\bar{w})$. Moreover, notice that SWF has increasing differences in $(\tau,\bar{w};\theta_H)$. In particular,

$$\frac{\partial^2\text{SWF}}{\partial\theta_H\partial\tau}=\frac{\alpha^3\theta_H(1-\tau)^{2\alpha-1}}{c_H^2\chi^{2\alpha}}>0,\ \text{and}\ \frac{\partial^2\text{SWF}}{\partial\theta_H\partial\bar{w}}=\frac{\chi\bar{w}^{(2-\alpha)/(\alpha-1)}}{(1-\alpha)\alpha^{1/\alpha+1}\theta_H^{1/\alpha-1}}>0.$$

Thus, by Topkis' Theorem,[19]

$$\frac{\partial\tau}{\partial\theta_H}>0,\ \text{and}\ \frac{\partial\bar{w}}{\partial\theta_H}>0.$$

This implies that $sgn\left(\frac{\partial\tau}{\partial\theta_H}\times\frac{\partial\bar{w}}{\partial\theta_H}\right)=1$, which completes the proof. $\square$

Proof of Lemma 7: Observe that Γ is a supermodular game as (i) the action sets $\{A_\tau,A_{\bar{w}}\}$ are compact, (ii) SWF is continuous in $(\tau,\bar{w})$, and (iii) SWF has increasing differences in the players own action and in each rival's action. If $\lambda>0$, then (iii) follows directly from Lemma 6(i). If $\lambda=0$, then

$$\frac{\partial^2\text{SWF}}{\partial\tau\partial\bar{w}}\mid_{\lambda=0}=\frac{\alpha}{1-\alpha}\bar{w}^{1/(\alpha-1)}\left[\frac{1}{c_L}+\frac{(1-\tau)}{c_L^2}\underbrace{\left(\frac{\theta_H\alpha}{\chi^\alpha}\left\{(1-\tau)^\alpha-\tau\alpha(1-\tau)^{\alpha-1}\right\}-\bar{w}^{\alpha/(\alpha-1)}\right)}_{=0\text{ from the optimality of }\tau}\right]>0.$$

Thus, by a theorem from Topkis (1979), B has extremal solutions that are increasing (co-

[19]Following Milgrom and Shannon (1994), this comparative statics analysis can be extended using only ordinal conditions.

ordinatewise) in each rival's action. The theorem characterizes the greatest and least element of the best response correspondence. However, since SWF is strictly concave (by Lemma 4), the greatest and least elements of the best response correspondence coincide. Nonemptyness of the best response correspondence follows from the continuity of SWF. Since $[0,1] \times [0, w_H]$ is compact, and B is an increasing function, by the Tarski (1955) fixed point theorem $\exists x \in [0,1] \times [0, w_H] : B(x) = x$, which completes the proof. $\square$

Derivation of the Social Welfare Function

A necessary condition for optimality under the assumed form of preferences is

$$\frac{w_H(1-\tau)}{c_H} = \chi. \tag{5}$$

Moreover, combining the government budget constraint, the binding budget constraint of high-skilled workers, and the labor demand condition for low-skilled workers, we have

$$c_H = w_H l_H (1 - \tau\mu_L) + \tau\mu_L \bar{w} \left(\frac{\bar{w}}{\theta_L \alpha} \right)^{1/(\alpha-1)}. \tag{6}$$

Under assumption 1, equation (6) reduces to

$$c_H \approx w_H l_H. \tag{7}$$

Substituting equation (7) in equation (5) we have

$$l_H \approx \frac{(1-\tau)}{\chi}. \tag{8}$$

The labor allocation of low-skilled workers is $l_L = \bar{w}^{1/(\alpha-1)}$. Substituting in the equilibrium labor allocations, equilibrium wages $w_L = \bar{w}$ and $w_H = \theta_H \alpha l_H^{\alpha-1}$, and assumption 1 in the low-skilled households budget constraint we get

$$c_L \approx \bar{w}^{\alpha/(\alpha-1)}(1-\tau) + \tau\theta_H\alpha \left\{ \frac{(1-\tau)}{\chi} \right\}^{\alpha}. \tag{9}$$

Lastly, substituting the equilibrium labor allocation and the equilibrium wage for the high-skilled in equation (7) we get

$$c_H \approx \theta_H \alpha \left\{ \frac{(1-\tau)}{\chi} \right\}^{\alpha}. \tag{10}$$

Thus, the social welfare function can be approximated by the maximand of (SWF).

Table 4: Individual Income Tax Returns Audited, by Size of Adjusted Gross Income, Fiscal Year 2017

Adjusted gross income	Returns filed in CY 2016 (% of total)	Audits in FY 2017 (%)
All returns	100.00	0.62
No adjusted gross income	1.69	2.55
$1 under $25,000	36.47	0.71
$25,000 under $50,000	23.33	0.49
$50,000 under $75,000	13.26	0.48
$75,000 under $100,000	8.59	0.45
$100,000 under $200,000	12.19	0.47
$200,000 under $500,000	3.60	0.70
$500,000 under $1,000,000	0.58	1.56
$1,000,000 under $5,000,000	0.26	3.52
$5,000,000 under $10,000,000	0.02	7.95
$10,000,000 or more	0.01	14.52

Notes: This table shows examination coverage of individual income tax returns classified by size of adjusted gross income. Adjusted gross income (AGI) is total income (including losses), as defined by the Internal Revenue Code, less statutory adjustments–primarily business, investment, and certain other deductions. Calendar Year (CY) 2016 data are presented because, in general, examination activity is associated with returns filed in the previous calendar year. The total number of individual income tax returns filed in CY 2016 was 149,919,416. The "No adjusted gross income" row includes returns with adjusted gross income of less than zero. AGI may be less than zero when a taxpayer reports losses or statutory adjustments that exceed total income. Observe that high-income tax payers are more likely to be audited. Interestingly, as a household's adjusted gross income approaches zero, it becomes slightly more likely to be subject to an IRS audit. This is because these low-income households are generally eligible for the EITC, a provision that is often claimed fraudulently or calculated incorrectly.
Source: IRS Data Book 2017.